

Λ and $\bar{\Lambda}$ polarization in polarized DISM. Anselmino¹, M. Boglione² and F. Murgia³¹ Dipartimento di Fisica Teorica, Università di Torino and
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C.P. 170, I-09042 Monserrato (CA), Italy**Abstract:**

We consider the polarization of $\Lambda + \bar{\Lambda}$ baryons produced in polarized Deep Inelastic Scattering at leading order, with various spin configurations: longitudinally polarized leptons and unpolarized nucleon; unpolarized leptons and longitudinally or transversely polarized nucleons; longitudinally polarized leptons and nucleons. We show how the different results in the different cases are related to different aspects of the elementary dynamics and to the spin properties of the distribution and fragmentation functions and show how a combined analysis might give useful information. We give numerical results according to several sets of polarized fragmentation functions recently proposed.

1. Introduction

Λ baryons produced in high energy interactions and resulting from quark fragmentation allow a unique test of spin transfer from partons to hadrons: the Λ polarization is easily measurable by looking at the angular distribution of the $\Lambda \rightarrow p\pi$ decay (in the Λ helicity rest frame) and the fragmenting parton polarization is determined by the elementary Standard Model interactions, provided one knows the initial parton spin state. In this respect Λ 's produced in lepton induced processes are particularly interesting and indeed several papers on this subject have recently been published or submitted to e-Print archives [1]-[9].

We perform here a detailed analysis of the polarization of Λ 's produced in polarized DIS; a general discussion of the helicity density matrix of hadrons produced in polarized lepton-nucleon interactions, at leading order, can be found in Refs. [10] and [1]. It can be written as:

$$\rho_{\lambda_h, \lambda'_h}^{(s, S)}(h) \frac{d\sigma^{\ell, s+N, S \rightarrow \ell+h+X}}{dx dy dz} = \sum_{q; \lambda_\ell, \lambda_q, \lambda'_q} \frac{1}{16\pi x s} \times \quad (1)$$

$$\rho_{\lambda_\ell, \lambda_\ell}^{\ell, s} \rho_{\lambda_q, \lambda'_q}^{q/N, S} f_{q/N}(x) \hat{M}_{\lambda_\ell, \lambda_q; \lambda_\ell, \lambda_q}^q \hat{M}_{\lambda_\ell, \lambda'_q; \lambda_\ell, \lambda'_q}^{q*} D_{\lambda_h, \lambda'_h}^{\lambda_q, \lambda'_q}(z),$$

where x and y are the usual DIS variables, $x = Q^2/2p \cdot q$, $y = Q^2/xs$ and, neglecting hadron masses, $z = p_h \cdot p/p \cdot q$, where p , q and p_h are, respectively, the nucleon, virtual photon and final hadron four-momenta. $\rho^{\ell, s}$ is the helicity density matrix of the initial lepton with spin s , $f_{q/N}(x)$ is the number density of unpolarized quarks q with momentum fraction x inside an unpolarized nucleon and $\rho^{q/N, S}$ is the helicity density matrix of quark q inside the polarized nucleon N with spin S . The $\hat{M}_{\lambda_\ell, \lambda_q; \lambda_\ell, \lambda_q}^q$'s are the helicity amplitudes for the elementary process $\ell q \rightarrow \ell q$. The final lepton spin is not observed and helicity conservation of perturbative QCD and QED has already been taken into account in the above equation: as a consequence only the diagonal elements of $\rho^{\ell, s}$ contribute to $\rho(h)$ and non diagonal elements, present in case of transversely polarized leptons, do not contribute. $D_{\lambda_h, \lambda'_h}^{\lambda_q, \lambda'_q}(z)$ is a generalized fragmentation function [10] related to the usual unpolarized fragmentation function $D_{h/q}(z)$, *i.e.* the density number of hadrons h resulting from the fragmentation of an unpolarized quark q and carrying a fraction z of its momentum, by

$$D_{h/q}(z) = \frac{1}{2} \sum_{\lambda_q, \lambda_h} D_{\lambda_h, \lambda_h}^{\lambda_q, \lambda_q}(z) = \frac{1}{2} \sum_{\lambda_q, \lambda_h} D_{h_{\lambda_h}/q_{\lambda_q}}(z), \quad (2)$$

where $D_{\lambda_h, \lambda_h}^{\lambda_q, \lambda_q}(z) \equiv D_{h_{\lambda_h}/q_{\lambda_q}}(z)$ is a polarized fragmentation function, *i.e.* the density number of hadrons h with helicity λ_h resulting from the fragmentation of a quark q with helicity λ_q . Angular momentum conservation and collinear configurations imply for the generalized fragmentation functions [10]:

$$D_{\lambda_h, \lambda'_h}^{\lambda_q, \lambda'_q} = 0 \quad \text{when} \quad \lambda_q - \lambda'_q \neq \lambda_h - \lambda'_h. \quad (3)$$

The elementary amplitudes \hat{M}^q are normalized so that the elementary unpolarized cross-section, $\ell q \rightarrow \ell q$, is:

$$\frac{d\hat{\sigma}^q}{dy} \equiv e_q^2 \frac{d\hat{\sigma}}{dy} = \frac{1}{16\pi x s} \frac{1}{4} \sum_{\lambda_\ell, \lambda_q} |\hat{M}_{\lambda_\ell, \lambda_q; \lambda_\ell, \lambda_q}^q|^2 = e_q^2 \frac{2\pi\alpha^2}{Q^2} \frac{1 + (1-y)^2}{y} . \quad (4)$$

Notice that there are only two independent helicity amplitudes:

$$\begin{aligned} \hat{M}_{++;++}^q = \hat{M}_{--;--}^q &\equiv e_q \hat{M}_{++;++} = e_q 8\pi\alpha \frac{1}{y} \\ \hat{M}_{+-;+-}^q = \hat{M}_{-+;-+}^q &\equiv e_q \hat{M}_{+-;+-} = e_q 8\pi\alpha \left[\frac{1}{y} - 1 \right] \end{aligned} \quad (5)$$

so that

$$\frac{d\hat{\sigma}^q}{dy} = \frac{1}{2} \left[\frac{d\hat{\sigma}_q^{++}}{dy} + \frac{d\hat{\sigma}_q^{+-}}{dy} \right] \quad (6)$$

with

$$\frac{d\hat{\sigma}_q^{++}}{dy} \equiv e_q^2 \frac{d\hat{\sigma}^{++}}{dy} = \frac{e_q^2}{16\pi x s} |\hat{M}_{++;++}|^2 = e_q^2 \frac{4\pi\alpha^2}{Q^2} \frac{1}{y} \quad (7)$$

and

$$\frac{d\hat{\sigma}_q^{+-}}{dy} \equiv e_q^2 \frac{d\hat{\sigma}^{+-}}{dy} = \frac{e_q^2}{16\pi x s} |\hat{M}_{+-;+-}|^2 = e_q^2 \frac{4\pi\alpha^2}{Q^2} \frac{(1-y)^2}{y} . \quad (8)$$

Finally, the cross-section appearing in the l.h.s. of Eq. (1), which gives the correct normalization to $\rho(h)$, $\text{Tr}\rho = 1$, can be written, using Eq. (3), as

$$\frac{d\sigma^{\ell, s+N, S \rightarrow \ell+h+X}}{dx dy dz} = \sum_{q; \lambda_\ell, \lambda_q} \frac{1}{16\pi x s} \rho_{\lambda_\ell, \lambda_\ell}^{\ell, s} \rho_{\lambda_q, \lambda_q}^{q/N, S} f_{q/N}(x) |\hat{M}_{\lambda_\ell, \lambda_q; \lambda_\ell, \lambda_q}^q(y)|^2 D_{h/q}(z) . \quad (9)$$

Eqs. (1)-(9) hold within QCD factorization theorem at leading twist and leading order in the coupling constants; the intrinsic \mathbf{k}_\perp of the partons have been integrated over and collinear configurations dominate both the distribution and the fragmentation functions. For simplicity of notations we have not indicated the Q^2 scale dependences in f and D .

We shall use such equations for spin 1/2 Λ baryons produced starting from several particular initial spin configurations; we will discuss how the measurable components of the Λ polarization vector depend on different combinations of distribution functions, elementary dynamics and fragmentation functions: each of these terms predominantly depends on a single variable, respectively x , y and z , and a careful analysis of different situations can yield precious information. Although Λ production in polarized DIS has been recently discussed in several papers, most of them only consider some initial spin configurations and specific models for fragmentation functions. Our analysis is more comprehensive and somewhat more general, emphasizing the physical meaning of possible measurements, and allowing also to obtain some general relationships between different polarization values.

2. Polarization vector of spin 1/2 baryons

We fix our spin notations in the $\ell - p$ center of mass frame: the lepton moves with four-momentum l along the z -axis and the proton moves with four-momentum p in the opposite direction; we choose xz as the lepton-hadron production plane, with the y -axis parallel to $\mathbf{l} \times \mathbf{p}_h$. We denote by S_L the (longitudinal) nucleon spin oriented along the z -axis, by S_S the (sideway) spin oriented along the x -axis and by S_N the (normal) spin oriented along the y -axis. Notice that $+S_L$ corresponds to a $-$ helicity proton and $-S_L$ to a $+$ helicity one; we will only consider longitudinally polarized leptons with spins $\pm s_L$ which correspond respectively to \pm helicities.

From Eqs. (1)-(9) one obtains the explicit expression for the components of the helicity density matrix of a spin 1/2 baryon. These are related to the three components of the baryon polarization vector, as measured in its helicity rest frame by $P_i(B) = \text{Tr}[(\sigma^i \rho(B)]$ ($i = x, y, z$); details can be found in Ref. [10]. We denote by (s, S) [or (h, H)] the (lepton, nucleon) spins [or helicities], 0 stands for unpolarized particle; one finds, for a spin 1/2 hadron B :

$$\begin{aligned} P_z^{(0, -S_L)}(B; x, y, z) &= P_z^{(0, +)}(B; x, y, z) = \frac{\sum_q \Delta q d\hat{\sigma}_q \Delta D_{B/q}}{\sum_q q d\hat{\sigma}_q D_{B/q}} \\ &= \frac{\sum_q e_q^2 \Delta q(x) \Delta D_{B/q}(z)}{\sum_q e_q^2 q(x) D_{B/q}(z)} \end{aligned} \quad (10)$$

$$\begin{aligned} P_z^{(s_L, 0)}(B; x, y, z) &= P_z^{(+, 0)}(B; x, y, z) = \frac{\sum_q (1/2) q [d\hat{\sigma}_q^{++} - d\hat{\sigma}_q^{+-}] \Delta D_{B/q}}{\sum_q q d\hat{\sigma}_q D_{B/q}} \\ &= \frac{\sum_q e_q^2 q(x) \Delta D_{B/q}(z)}{\sum_q e_q^2 q(x) D_{B/q}(z)} \hat{A}_{LL}(y) \end{aligned} \quad (11)$$

$$\begin{aligned} P_y^{(0, S_N)}(B; x, y, z) &= -P_x^{(0, S_S)}(B; x, y, z) = \frac{\sum_q \Delta_T q [d\hat{\sigma}_q^{\uparrow \rightarrow \uparrow} - d\hat{\sigma}_q^{\uparrow \rightarrow \downarrow}] \Delta_T D_{B/q}}{\sum_q q d\hat{\sigma}_q D_{B/q}} \\ &= \frac{\sum_q e_q^2 \Delta_T q(x) \Delta_T D_{B/q}(z)}{\sum_q e_q^2 q(x) D_{B/q}(z)} \hat{D}_{NN}(y) \end{aligned} \quad (12)$$

$$\begin{aligned} P_z^{(s_L, -S_L)}(B; x, y, z) &= P_z^{(+, +)}(B; x, y, z) = \frac{\sum_q [q_+ d\hat{\sigma}_q^{++} - q_- d\hat{\sigma}_q^{+-}] \Delta D_{B/q}}{\sum_q [q_+ d\hat{\sigma}_q^{++} + q_- d\hat{\sigma}_q^{+-}] D_{B/q}} \\ &= \frac{\sum_q e_q^2 [q(x) \hat{A}_{LL}(y) + \Delta q(x)] \Delta D_{B/q}(z)}{\sum_q e_q^2 [q(x) + \hat{A}_{LL}(y) \Delta q(x)] D_{B/q}(z)} \end{aligned} \quad (13)$$

$$\begin{aligned} P_z^{(s_L, +S_L)}(B; x, y, z) &= P_z^{(+, -)}(B; x, y, z) = \frac{\sum_q [q_- d\hat{\sigma}_q^{++} - q_+ d\hat{\sigma}_q^{+-}] \Delta D_{B/q}}{\sum_q [q_- d\hat{\sigma}_q^{++} + q_+ d\hat{\sigma}_q^{+-}] D_{B/q}} \\ &= \frac{\sum_q e_q^2 [q(x) \hat{A}_{LL}(y) - \Delta q(x)] \Delta D_{B/q}(z)}{\sum_q e_q^2 [q(x) - \hat{A}_{LL}(y) \Delta q(x)] D_{B/q}(z)} \end{aligned} \quad (14)$$

where $d\hat{\sigma}_q$ stands for $d\hat{\sigma}_q/dy$.

Some comments are in order.

- The above results are known [1, 2, 11]; we have rederived and grouped them here for convenience and further discussion. Notice that all other spin configurations – at leading order – either give the same results or no polarization. We remind that $q_\lambda = f_{q_\lambda/p_+}$ is the number density of quarks with helicity λ inside a $+$ helicity proton, $q(x) = q_+(x) + q_-(x)$ and $\Delta q(x) = q_+(x) - q_-(x)$. Similarly $\Delta D_{B/q} = D_{B_+/q_+} - D_{B_-/q_-}$; $\Delta_T q$ and $\Delta_T D_{B/q}$ are respectively the analogue of Δq and $\Delta D_{B/q}$ for transverse spins.
- The longitudinal B polarization induced by a longitudinal nucleon polarization, Eq. (10), *does not* depend on the elementary dynamics, but only on the quark spin distribution and fragmentation properties. Neglecting the QCD Q^2 -evolution, $P_z^{(0,-S_L)}$ does not depend on the DIS variable y , but only on x and z .
- The longitudinal B polarization resulting from the scattering of longitudinally polarized leptons off unpolarized nucleons depends on the unpolarized distribution functions, the polarized fragmentation functions and the elementary dynamics, through the double spin asymmetry for the $\ell q \rightarrow \ell q$ process [see Eqs. (6)-(8)]:

$$\hat{A}_{LL}(y) = \frac{d\hat{\sigma}_q^{++} - d\hat{\sigma}_q^{+-}}{d\hat{\sigma}_q^{++} + d\hat{\sigma}_q^{+-}} = \frac{d\hat{\sigma}_q^{++} - d\hat{\sigma}_q^{+-}}{2 d\hat{\sigma}_q} = \frac{y(2-y)}{1 + (1-y)^2} . \quad (15)$$

Notice that \hat{A}_{LL} grows with y from 0 (at $y = 0$) to 1 (at $y = 1$), so that $P_z^{(s_L,0)}$ is an *increasing* function of y , starting from 0 at $y = 0$.

- The transverse B polarization induced by a transverse nucleon polarization, Eq. (12), depends on the quark transverse spin distribution and fragmentation properties and on the elementary dynamics, through the double transverse spin asymmetry for the $\ell q \rightarrow \ell q$ process:

$$\begin{aligned} \hat{D}_{NN}(y) &= \frac{d\hat{\sigma}^{\ell q^\uparrow \rightarrow \ell q^\uparrow} - d\hat{\sigma}^{\ell q^\uparrow \rightarrow \ell q^\downarrow}}{d\hat{\sigma}^{\ell q^\uparrow \rightarrow \ell q^\uparrow} + d\hat{\sigma}^{\ell q^\uparrow \rightarrow \ell q^\downarrow}} \\ &= \frac{d\hat{\sigma}^{\ell q^\uparrow \rightarrow \ell q^\uparrow} - d\hat{\sigma}^{\ell q^\uparrow \rightarrow \ell q^\downarrow}}{d\hat{\sigma}_q} = \frac{2(1-y)}{1 + (1-y)^2} \end{aligned} \quad (16)$$

where $\uparrow = S_N$ and $\downarrow = -S_N$.

Contrary to \hat{A}_{LL} , \hat{D}_{NN} decreases with y , with $\hat{D}_{NN} = 1$ at $y = 0$ and $\hat{D}_{NN} = 0$ at $y = 1$; thus, $P_y^{(0,S_N)}$ is a *decreasing* function of y , reaching 0 at $y = 1$. An experimental confirmation of the opposite y -dependences of $P_z^{(s_L,0)}$ and $P_y^{(0,S_N)}$ would supply a new, subtle and important test of the factorization scheme of Eq. (1). \hat{D}_{NN} is the transverse polarization of the final quark generated by an initial transversely polarized (\uparrow) quark in the $\ell q \rightarrow \ell q$ process, and it is usually referred to as the depolarization factor.

- When both the lepton and the nucleon are longitudinally polarized the B resulting polarization depends on yet a different combination of polarized quark distribution functions, fragmentation functions and elementary dynamics. It is then clear why a combined study of $P(B)$ in different cases could yield unique information.
- Finally, we notice that, in general, $P_i^{(s,S)} = -P_i^{(-s,-S)}$.

3. Polarization vector of Λ baryons

We now consider the particular case of Λ baryons and discuss possible ways of extracting information from combined measurements of $P_i(\Lambda)$; as we said the Λ polarization vector can be measured by looking at the proton angular distribution as resulting from $\Lambda \rightarrow \pi p$ decay in the Λ helicity rest frame, that is the frame obtained by rotating the $\ell - p$ c.m. frame around the y -axis so that the new z_h -axis is parallel to the Λ direction, and then boosting along z_h with the same speed as the Λ :

$$\begin{aligned} W(\theta_p, \phi_p) &= \frac{1}{4\pi} [1 + \alpha(P_z \cos \theta_p + P_x \sin \theta_p \cos \phi_p + P_y \sin \theta_p \sin \phi_p)] \\ &= \frac{1}{4\pi} [1 + \alpha \mathbf{P} \cdot \hat{\mathbf{p}}] \end{aligned} \quad (17)$$

where $\alpha = 0.642 \pm 0.013$.

We follow Ref. [3] and assume for the unpolarized fragmentation functions:

$$D_{\Lambda/u} = D_{\Lambda/d} = D_{\Lambda/s} = D_{\Lambda/\bar{u}} = D_{\Lambda/\bar{d}} = D_{\Lambda/\bar{s}} \equiv D_{\Lambda/q} \quad (18)$$

where Λ means $\Lambda^0 + \bar{\Lambda}^0$.

The heavy quark and gluon unpolarized fragmentation functions play a negligible role for $z \gtrsim 0.3$ [3] and we neglect them here: we have actually checked that our results, when comparable, are almost indistinguishable from those of Ref. [3] where also heavy quark and gluon contributions to the unpolarized cross-sections are taken into account.

Similarly, we follow Ref. [3] for the polarized fragmentation functions:

$$\Delta D_{\Lambda/u}(z, Q_0^2) = \Delta D_{\Lambda/d}(z, Q_0^2) = N_u \Delta D_{\Lambda/s}(z, Q_0^2). \quad (19)$$

Eqs. (19) holds also for light antiquarks and it remains valid through QCD Q^2 -evolution; heavy quark contributions are neglected.

Using Eq. (18) and (19) into Eqs. (10)-(11) and (13)-(14) gives:

$$P_z^{(0,+)}(\Lambda; x, y, z) = \frac{\Delta Q'(x)}{Q(x)} \frac{\Delta D_{\Lambda/s}(z)}{D_{\Lambda/q}(z)} \quad (20)$$

$$P_z^{(+,0)}(\Lambda; x, y, z) = \frac{Q'(x)}{Q(x)} \frac{\Delta D_{\Lambda/s}(z)}{D_{\Lambda/q}(z)} \hat{A}_{LL}(y) \quad (21)$$

$$P_z^{(+,+)}(\Lambda; x, y, z) = \frac{Q'(x) \hat{A}_{LL}(y) + \Delta Q'(x)}{Q(x) + \Delta Q(x)} \frac{\Delta D_{\Lambda/s}(z)}{D_{\Lambda/q}(z)} \quad (22)$$

$$P_z^{(+,-)}(\Lambda; x, y, z) = \frac{Q'(x) \hat{A}_{LL}(y) - \Delta Q'(x)}{Q(x) - \Delta Q(x)} \frac{\Delta D_{\Lambda/s}(z)}{D_{\Lambda/q}(z)} \quad (23)$$

where

$$Q \equiv 4(u + \bar{u}) + (d + \bar{d}) + (s + \bar{s}) \quad (24)$$

$$\Delta Q \equiv 4(\Delta u + \Delta \bar{u}) + (\Delta d + \Delta \bar{d}) + (\Delta s + \Delta \bar{s}) \quad (25)$$

$$Q' \equiv [4(u + \bar{u}) + (d + \bar{d})] N_u + (s + \bar{s}) \quad (26)$$

$$\Delta Q' \equiv [4(\Delta u + \Delta \bar{u}) + (\Delta d + \Delta \bar{d})] N_u + (\Delta s + \Delta \bar{s}) . \quad (27)$$

If one assumes the same relation (19) to hold also for transversely polarized quark fragmentation functions, then Eq. (12) yields:

$$P_y^{(0,S_N)}(\Lambda; x, y, z) = \frac{\Delta_T Q'(x)}{Q(x)} \frac{\Delta_T D_{\Lambda/s}(z)}{D_{\Lambda/q}(z)} \hat{D}_{NN}(y) \quad (28)$$

with

$$\Delta_T Q' \equiv [4(\Delta_T u + \Delta_T \bar{u}) + (\Delta_T d + \Delta_T \bar{d})] N_u + (\Delta_T s + \Delta_T \bar{s}) . \quad (29)$$

Eqs. (20)-(23) hold under assumptions (18) and (19) alone, independently of the actual value of N_u ; as the polarized and unpolarized distribution functions are well known and several sets are available in the literature, we can exploit Eqs. (20) and (21) to obtain:

$$N_u = -\frac{S}{U} \frac{\hat{A}_{LL} P_z^{(0,+)} - (\Delta S/S) P_z^{(+,0)}}{\hat{A}_{LL} P_z^{(0,+)} - (\Delta U/U) P_z^{(+,0)}} \quad (30)$$

and

$$\frac{\Delta D_{\Lambda/s}(z)}{D_{\Lambda/q}(z)} = \frac{Q}{S} \frac{1}{\hat{A}_{LL}} \frac{(\Delta U/U) P_z^{(+,0)} - \hat{A}_{LL} P_z^{(0,+)}}{(\Delta U/U) - (\Delta S/S)} \quad (31)$$

where we have defined

$$U \equiv 4(u + \bar{u}) + (d + \bar{d}) \quad S \equiv s + \bar{s} \quad \Delta U \equiv 4(\Delta u + \Delta \bar{u}) + (\Delta d + \Delta \bar{d}) \quad \Delta S \equiv \Delta s + \Delta \bar{s}. \quad (32)$$

Eqs. (22) and (23) can then be used to predict – within the general assumptions (18) and (19) – the following interesting relations between polarization observables:

$$P_z^{(+,\pm)} = \frac{P_z^{(+,0)} \pm P_z^{(0,+)}}{1 \pm (\Delta Q/Q) \hat{A}_{LL}} \quad (33)$$

and

$$P_z^{(+,+)} - P_z^{(+,-)} = 2 \frac{P_z^{(0,+)} - (\Delta Q/Q) \hat{A}_{LL} P_z^{(+,0)}}{1 - [(\Delta Q/Q) \hat{A}_{LL}]^2} . \quad (34)$$

Notice that in the small y region, due to the elementary dynamics, see Eq. (15), one has:

$$P_z^{(+,0)} \simeq 0 \quad P_z^{(+,\pm)} \simeq \pm P_z^{(0,+)} \quad (y \ll 1). \quad (35)$$

In the large y region instead, again from Eq. (15) and from the fact that large y implies small x , where $Q(x) \gg \Delta Q(x)$ and $Q'(x) \gg \Delta Q'(x)$, we expect:

$$P_z^{(+,+)} \simeq P_z^{(+,-)} \simeq P_z^{(+,0)} \quad (y \simeq 1). \quad (36)$$

We conclude this Section by reminding that we have derived our results for $\Lambda = \Lambda^0 + \bar{\Lambda}^0$, which allows assumption (18) concerning q and \bar{q} fragmentation functions. However, Eqs. (20)-(23) and (28) hold identical also for single Λ^0 production, provided one neglects the fragmentation function of a \bar{q} into Λ^0 , *i.e.* one neglects all \bar{q} terms in Eqs. (24)-(27) and (29). Anyway, the production of $\bar{\Lambda}^0$, in ℓp processes is strongly suppressed by the limited amount of initial \bar{q} , unless one considers very small x values.

4. Numerical estimates

We give now some numerical estimates of Eqs. (20)-(23) and (28); we use the sets of unpolarized and polarized fragmentation functions introduced and discussed by the authors of Ref. [3]: together with Eqs. (18), we use the expression for the unpolarized fragmentation functions they obtained by fitting $e^+e^- \rightarrow \Lambda X$ data. At initial $Q_0^2 = 0.23 \text{ (GeV}/c)^2$ scale one has, from a leading order (LO) analysis [3]:

$$D_{\Lambda/q}(z, Q_0^2) = 0.63 z^{0.23} (1 - z)^{1.83}. \quad (37)$$

The polarized fragmentation functions are assumed [3] to be of the initial form (19), with:

$$\Delta D_{\Lambda/s}(z, Q_0^2) = z^\alpha D_{\Lambda/q}(z, Q_0^2). \quad (38)$$

Leading order QCD evolution is consistently taken into account in our numerical computations.¹ Next to leading order contributions to the Λ polarization have been shown to be tiny [3] and we neglect them.

The parameter N_u defined in Eq. (19), has been chosen according to three different scenarios typical of a wide range of plausible models, and the corresponding remaining parameter α of Eq. (38) has been fixed by fitting the few LEP data on Λ polarization, with the results [3]:

- 1) **$N_u = 0$, $\alpha = 0.62$.** This scenario corresponds to $SU(6)$ non relativistic quark model, according to which the whole Λ spin is carried by the s quark.
- 2) **$N_u = -0.2$, $\alpha = 0.27$.** Such value of N_u is suggested in Ref. [12], based on a $SU(3)$ flavour symmetry analysis and on data on the first moment of g_1^p .

¹We are very grateful to D. de Florian, M. Stratmann and W. Vogelsang for providing us with their FORTRAN package for unpolarized and polarized fragmentation functions.

- 3) $N_u = 1$, $\alpha = 1.66$. A scenario in which, contrary to the non relativistic models, all light quarks contribute equally to the Λ polarization.

The unpolarized and polarized distribution functions are taken respectively from Refs. [13] and [14]. We have explicitly checked that a different choice of unpolarized and polarized distribution functions, like those of Refs. [15] and [16], can change significantly the numerical values of $P_i(\Lambda)$ (but not their qualitative behaviour) only for $x \gtrsim 0.3$.

A computation of $P_y^{(0,S_N)}$, Eq. (28), requires the knowledge of the quark transversely polarized distributions, $\Delta_T q$ or h_{1q} , and of the transversely polarized fragmentation functions, $\Delta_T D$, which are not known. In order to give an estimate we fix $\Delta_T q$ for u and d quarks by saturating the Soffer's bound [17] (assuming the same signs for $\Delta_T q$ and Δq):

$$\Delta_T u = \frac{1}{2} (u + \Delta u) \quad \Delta_T d = -\frac{1}{2} (d + \Delta d) . \quad (39)$$

All other transverse distributions ($\Delta_T \bar{q}$ and $\Delta_T s$) are neglected here and we also assume $\Delta_T D_{\Lambda/s} = \Delta D_{\Lambda/s}$.

A sample of typical results is presented in Figs. 1-4, for HERMES kinematics, $s = 52.4 \text{ (GeV)}^2$ and $Q^2 \gtrsim 1 \text{ (GeV}/c)^2$.

- In Fig. 1a we plot $P_z^{(0,+)}$ – at fixed $x = 0.1$ and $z = 0.5$ values – as a function of y , for each of the three scenarios; $P_z^{(0,+)}$, Eq. (20), can depend on y only via the Q^2 -evolution and indeed the three curves show an almost flat behaviour. The three scenarios yield quite different results. The minimum value of y is given by $y_{min} = Q_{min}^2/(xs) \simeq 0.19$.

The same plot for $P_z^{(+,0)}$ is presented in Fig. 1b; the y -dependence is essentially due to the factor \hat{A}_{LL} in Eq. (21); scenario 3, which assumes $Q'(x) = Q(x)$, together with Eq. (38) in which we neglect the mild Q^2 -evolution (taken into account in our numerical computations), gives a particularly simple result:

$$P_z^{(+,0)} \simeq z^{1.66} \hat{A}_{LL} . \quad (40)$$

- $P_z^{(+,+)}$ and $P_z^{(+,-)}$, again as functions of y at fixed $x = 0.1$ and $z = 0.5$ values, are shown in Figs. 2a and 2b respectively. At large $y \rightarrow 1$ values Eq. (36) is satisfied. The small $y \rightarrow 0$ behaviour cannot be seen with $x = 0.1$; in Fig. 2c and 2d we plot respectively $P_z^{(+,+)}$ and $P_z^{(+,-)}$, changing the x value to $x = 0.3$ and keeping $z = 0.5$. The allowed minimum value of y is now 0.06 and we have checked that Eq. (35) is indeed obeyed (by comparing with $P_z^{(0,+)}$ as a function of y at $x = 0.3$ and $z = 0.5$, not shown in Fig. 1); the change in sign of $P_z^{(+,-)}$ is particularly interesting.
- In Figs. 3a-3d we show respectively $P_z^{(0,+)}$, $P_z^{(+,0)}$, $P_z^{(+,+)}$ and $P_z^{(+,-)}$, at fixed values of $Q^2 = 1.7 \text{ (GeV}/c)^2$ and $z = 0.5$, as functions of x . At fixed Q^2 , y

decreases with increasing x – and viceversa – and this explains why relations (35) and (36) hold respectively when $x \rightarrow 1$ and $x \rightarrow 0$. Once more, the three different scenarios give very different results. Using different sets of polarized distribution functions, like [15] or [16] instead of [14], gives almost identical results for $x \lesssim 0.3$ and larger (in magnitude) results for $x \gtrsim 0.3$ (of course, $P_z^{(+,0)}$ is not affected at all by a change in Δq).

- In Figs. 4a we plot $P_y^{(0,S_N)}$, Eq. (28), at fixed $x = 0.1$ and $z = 0.5$ values, as a function of y for all scenarios; the y dependence is almost entirely given by \hat{D}_{NN} , Eq. (16), and indeed $P_y^{(0,S_N)} \rightarrow 0$ when $y \rightarrow 1$. Notice the opposite y behaviour of $P_z^{(+,0)}$ and $P_y^{(0,S_N)}$ due to the opposite behavior of \hat{A}_{LL} and \hat{D}_{NN} . In Fig. 4b $P_y^{(0,S_N)}$ is plotted as a function of x at fixed $Q^2 = 1.7 \text{ (GeV/c)}^2$ and $z = 0.5$.
- In general one finds very small values of $P_i(\Lambda)$ in scenario 1, negative values in scenario 2 and positive ones in scenario 3. This can easily be understood by the different values of N_u in the three scenarios, which assign respectively zero, negative, and positive contributions to u and d quarks, which dominate in the proton. Experimental measurements can easily discriminate between them.
- Recently HERMES Collaboration [18] have published a single experimental measurement of $P_z^{(+,0)}/\hat{A}_{LL}$, as a function of z and this seems to favour the scenario 1 prediction of Ref. [3], although errors and uncertainties are still too large to draw any reliable conclusions. Similarly, two values of $P_z^{(+,0)}(z)$ published by the E665 Collaboration [19] still have much too large errors.

5. Conclusions

The study of the angular distribution of the $\Lambda \rightarrow p\pi$ decay allows a simple and direct measurement of the components of the Λ polarization vector. For Λ 's produced in the current fragmentation region in DIS processes, the component of the polarization vector are related to spin properties of the quark inside the nucleon, to spin properties of the quark hadronization, and to spin dynamics of the elementary interactions. All this information, concerning quark distribution functions, quark fragmentation functions and spin properties of elementary dynamics are essentially factorized and separated as depending on three different variables, respectively x , z and y . The Q^2 -evolution and dependence of distribution and fragmentation functions somewhat mix the three variables, but smoothly, keeping separated the main properties of each of the different aspects of the process. Moreover, such Q^2 dependence is perturbatively well known and under control.

We have discussed all different polarization states of baryons, obtainable in the fragmentation of a quark in DIS with polarized initial leptons and nucleons, Eqs.

(10)-(14), showing how they can reveal different quark features, weighted and shaped by elementary dynamics.

Adopting a simplifying – although rather general and representative of many possible choices – assumption about the quark fragmentation functions into a Λ [3], we are able to extract from measurements further information on the quark fragmentation process, Eqs. (30) and (31), and to predict relations among polarization states induced by different initial spin configurations, Eqs. (33) and (34).

Numerical estimates are given in Figs. 1-4, according to three largely different scenarios [3] for fragmentation functions; each scenario has physical motivations and yields qualitatively different results: compatible with zero, large and negative, large and positive. Such results are stable against different choices of the polarized and unpolarized distribution functions, so that experimental information should immediately allow to draw clear conclusions and to learn about quark fragmentation properties.

The elementary dynamics fixes the small or large x or y behaviour of some of the polarization components; although expected, such behaviours should indeed be checked, as an independent and non trivial test of the QCD factorization scheme of Eq. (1); such a scheme has been widely used and tested for the computation of semi-inclusive unpolarized cross-sections, but not for more subtle spin observables.

We think that our comparative and comprehensive discussion of all possible Λ polarization measurements in polarized DIS is useful and can lead to a new and clear strategy which allows to obtain novel information.

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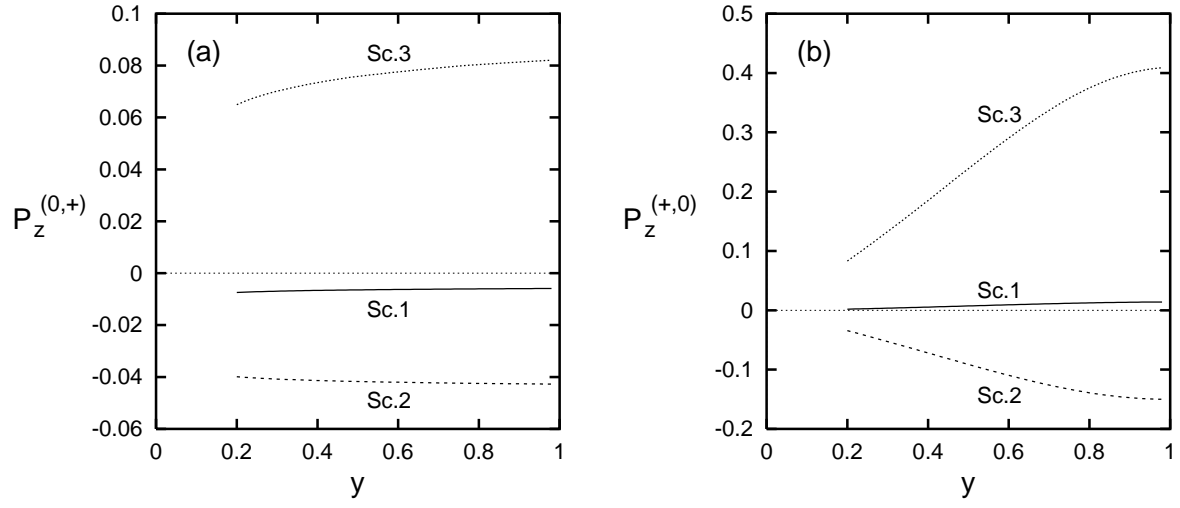


Figure 1: $P_z^{(0,+)}$ and $P_z^{(+,0)}$ as a function of y at fixed values of $x = 0.1$ and $z = 0.5$, for the three different scenarios.

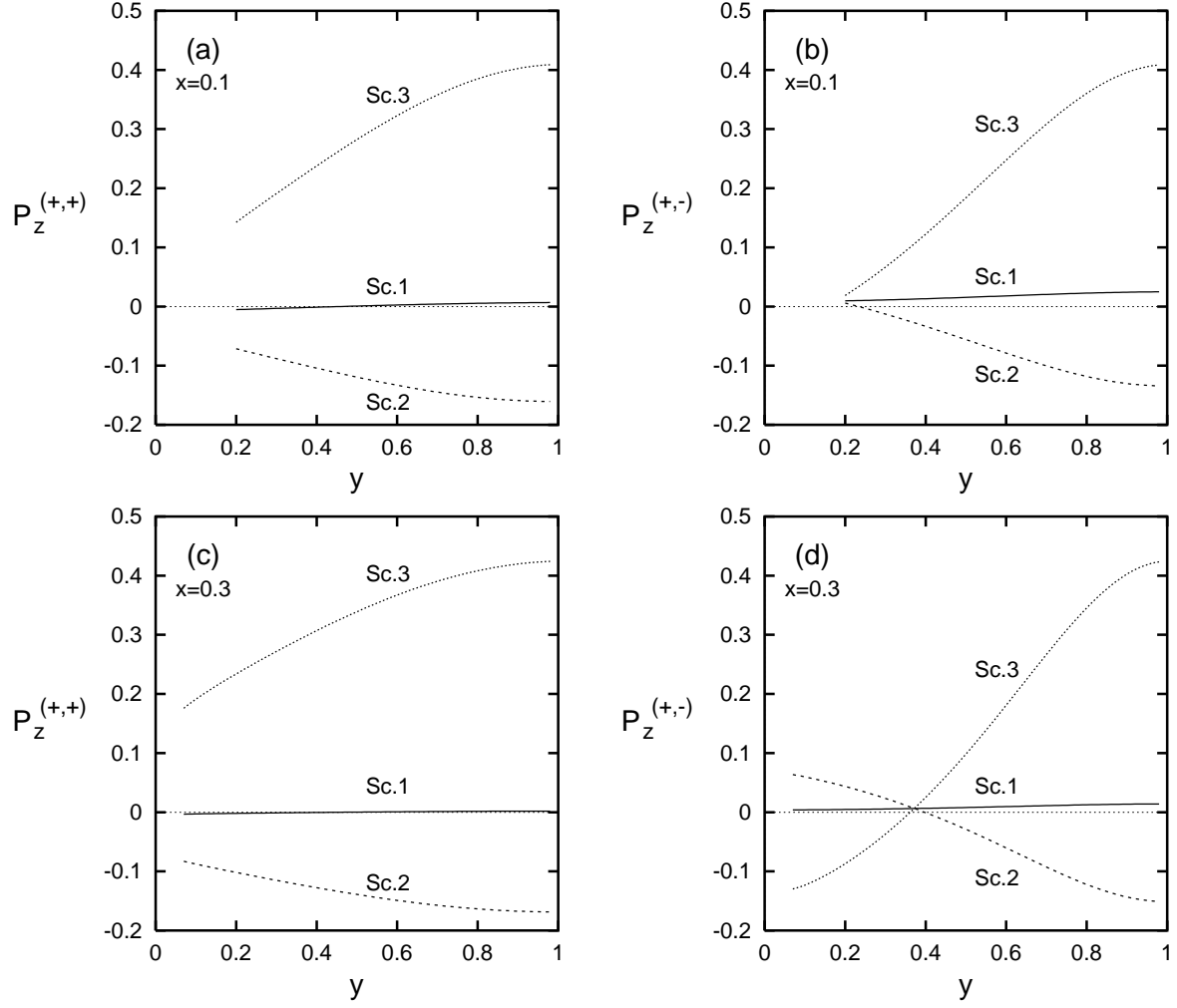


Figure 2: $P_z^{(+,+)}$ and $P_z^{(+,-)}$ as a function of y at fixed values of $z = 0.5$ for the three different scenarios. The upper plots correspond to fixed $x = 0.1$ and the lower plots to $x = 0.3$.

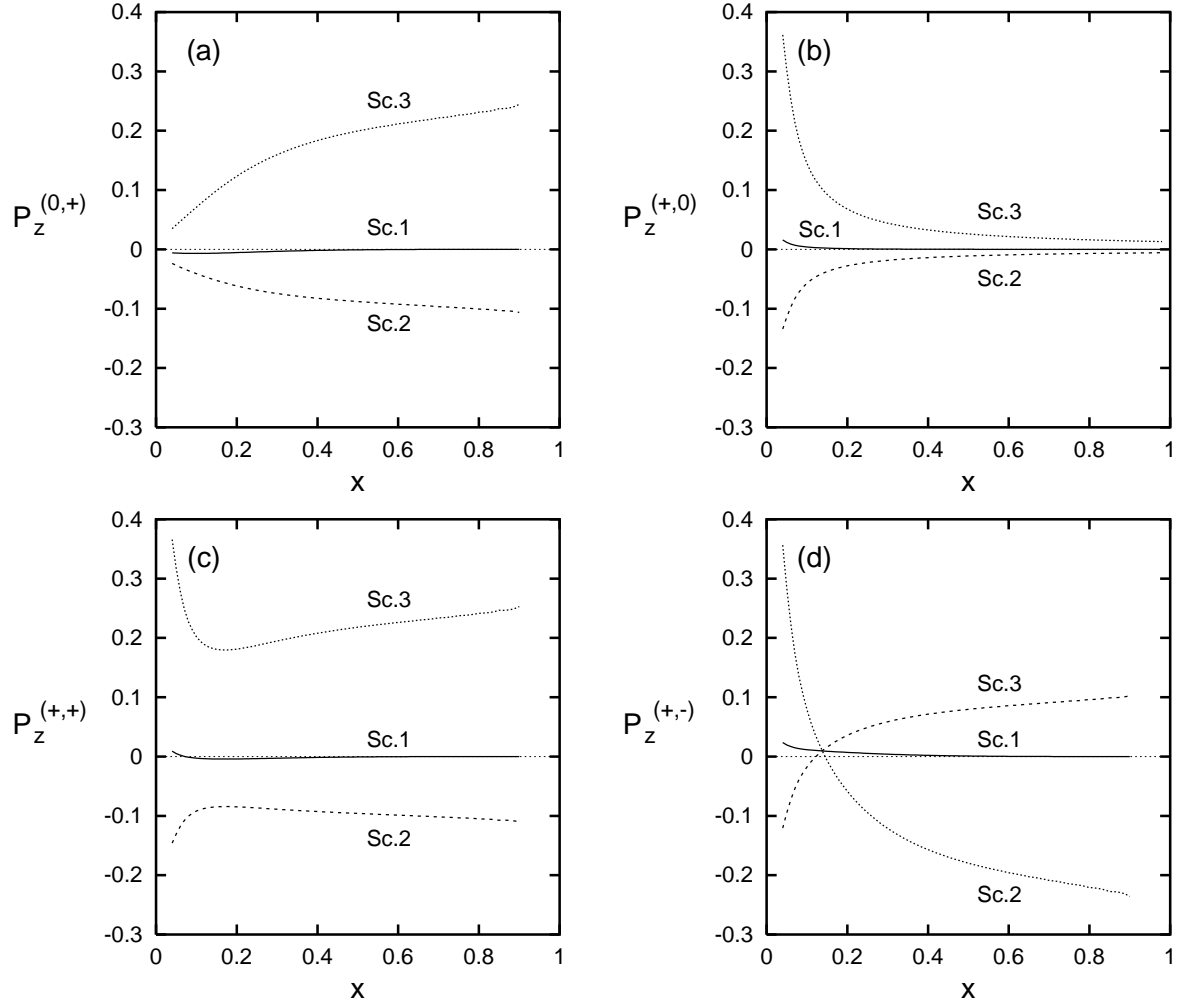


Figure 3: $P_z^{(0,+)}$, $P_z^{(+,0)}$, $P_z^{(+,+)}$ and $P_z^{(+,-)}$ as a function of x at fixed values of $Q^2 = 1.7 \text{ (GeV/c)}^2$ and $z = 0.5$, for the three different scenarios.

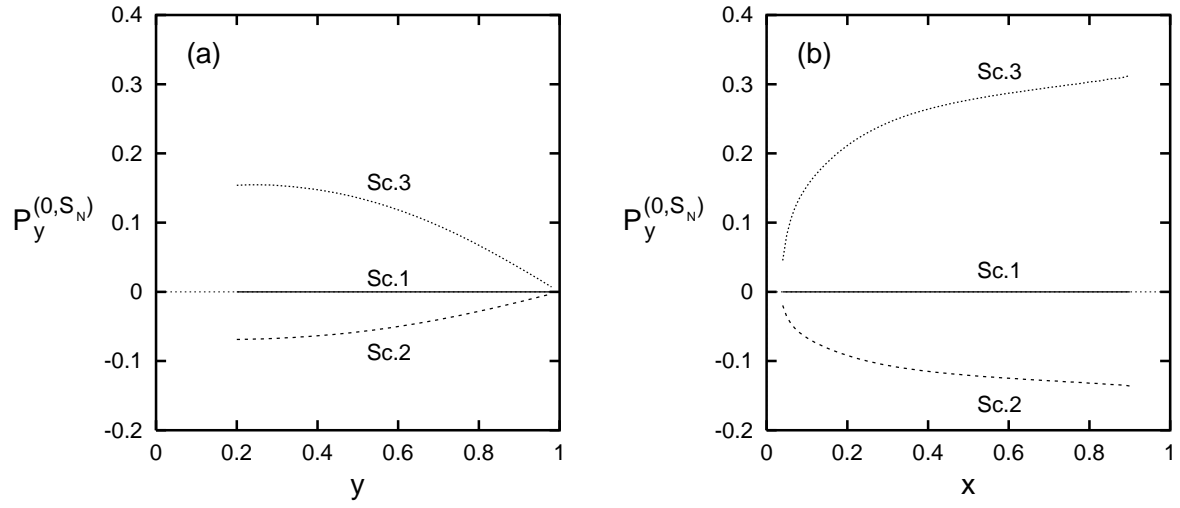


Figure 4: (a), $P_y^{(0,S_N)}$ as a function of y at fixed $x = 0.1$ and $z = 0.5$; (b), $P_y^{(0,S_N)}$ as a function of x at fixed $Q^2 = 1.7 \text{ (GeV/c)}^2$ and $z = 0.5$.